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ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains exercises on graphing equations with lower and upper brackets, a summary of the problems from the film "Graphing with Square Brackets," and the supplement.
(MK)

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THE ARITHMETIC PROJECT COURSE FOR TEACHERS

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TOPIC: Graphing Equations With Lower
and Upper Brackets.

FILM: Graphing With Square Brackets,
Grade 5

SUPPLEMENT: Graphing Number Line Jumping
Rules, Part One

NAME:

13

SE 031 113

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BOOK THIRTEEN

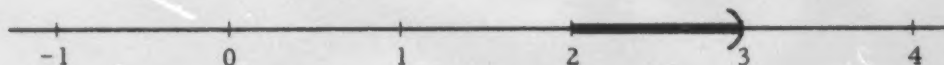
(Note: You may wish to refer to Books 7 and 8 before working this lesson.)

I. GRAPHING WITH LOWER AND UPPER BRACKETS

Instead of describing the numbers that work in a particular equation, you can show them on a number line. Some equations with lower or upper brackets give regions on the number line. Sample:

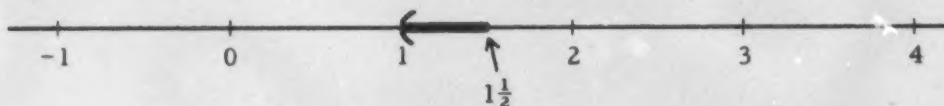
$$4 + \boxed{} = 6$$

The numbers that work are all the numbers from 2 on up to but not including 3. They may be shown on the number line this way. The curved line shows that 3 is not included.



Another sample:

$$\boxed{2 \times } = 3$$



Now do these:

1. $\boxed{2 \times } = 6$



2. $\left[2 \times \square \right] = 6$

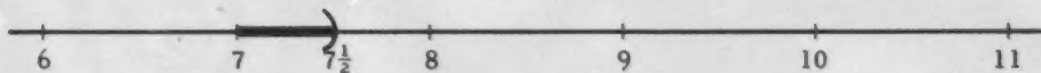


3. $\left[2 \times (\square - 5) \right] = 6$



Compare the answers in your last two problems.

4. Make up an equation so that the region indicated on the graph will be the answer.



Graph the numbers that work:

5. $\left[3 \times \square \right] = 6$



6. $\left[3 \times (\square - 4) \right] = 6$



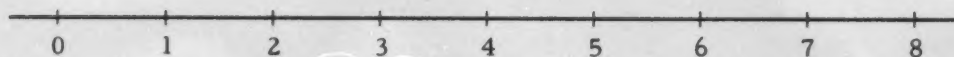
$$7. \quad \left[3 \times \square - 15 \right] = 6$$



$$8. \quad \left[3 \times \square - 15 \right] = 6$$



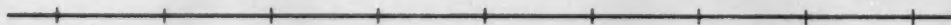
$$9. \quad 10 + \left[\frac{\square}{2} \right] = 11$$



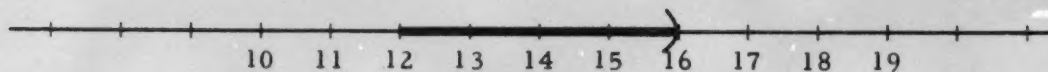
$$10. \quad \left[\frac{\square}{2} \right] = 3$$



$$11. \quad \left[\frac{\square}{5} \right] = 2$$



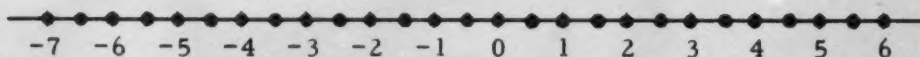
12. Write an equation that works for the given region.



II.

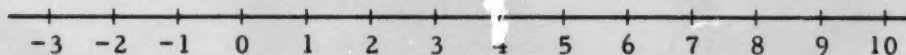
Here are equations and number lines. Put a dot or dots on the number line for the number or numbers that work.

Example: $\left[2 \times \square \right] = 2 \times \square$



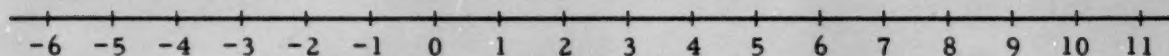
This graph shows that when you put 1 , $3\frac{1}{2}$, $-2\frac{1}{2}$, etc., in the boxes, the equation is true. In all of these problems, the dots on the number line continue indefinitely in both directions. Thus 15 , $100\frac{1}{2}$, and -37 also work. But $2\frac{1}{4}$, $8\frac{2}{3}$, and $-5\frac{2}{3}$ do not work.

1. $\left[\square \div 2 \right] = \square \div 2$



If students have trouble with a problem like this, ask them questions such as, "If I put 6 in the box, does it work? Does 8 work? Does $8\frac{1}{4}$ work? Does 7 work?" Do not forget to see whether 0 works.

2. $\left[\square \div 5 \right] = \square \div 5$



3. $\left[\square \div 10 \right] = \square \div 10$

(You calibrate the number line below so as to picture your answer adequately.)

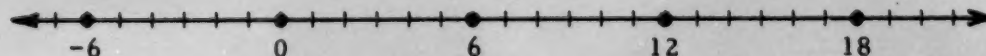


- ☆4. Calibrate the number line and put in the dots for the numbers that work.

$\left[(\square - 2) \div 10 \right] = (\square - 2) \div 10$



- ☆5. The pattern of the dots on the number line below continues in both directions. Make up an equation so that these dots (and only these dots) will work.



☆6.

Write an equation so that your dots will be three units apart. Calibrate the number line and put in the dots for the numbers that work for your equation.



☆7.

Write an equation so that your dots will be three units apart, and so that one of the dots is at 11. Calibrate the number line and put in the dots for the numbers that work for your equation.



III.

Assume you teach a fifth or sixth grade, and you have worked with problems involving the multiplication of negatives (i.e., your class has done such things as $-4 \times -4 = 16$).

Now consider the rule

$$\square \longrightarrow \square \times \square$$

Write 10 or more specific questions or problems (and give the answers) that you might use to enable your students to make some generalizations about this rule. You want them to find out such things as these: From what starting points do jumps with this rule go to the right? To the left? Are there regions you can't land on at all? What about standstill points?

Summary of Problems in the Film
"Graphing With Square Brackets"

5th Grade, James Russell Lowell School, Watertown, Massachusetts
Teacher: David A. Page

(Note: This film was made before the Project adopted the lower and upper brackets notation ($\lfloor$ $\rfloor$ and $\lceil$ $\rceil$). This summary uses lower brackets, $\lfloor$ $\rfloor$, which means exactly the same as square brackets, $[$ $]$.)

$$\lfloor 5\frac{3}{4} \rfloor =$$

$$\lfloor 6 + 3\frac{9}{10} \rfloor =$$

$$\lfloor 6\frac{1}{2} + 3\frac{9}{10} \rfloor =$$

(10)

Why?

("6 + 3 is 9, and $\frac{9}{10}$ is already $\frac{1}{2}$, and $\frac{1}{2}$ and $\frac{1}{2}$ is 1, and $\frac{4}{10}$ left is no good.")

$$\lfloor 6\frac{1}{2} \rfloor + \lfloor 3\frac{9}{10} \rfloor =$$

$$\lfloor \square \div 3 \rfloor = 4\frac{1}{2}$$

(Answers given by students were "impossible" and " $12\frac{3}{6}$ " .)

Who has a number that works?

(Even if it did come out to $4\frac{1}{2}$, the square brackets makes it 4.)

$$\lfloor \square \div 3 \rfloor = 4$$

Tell me a number that works. Let's try 10.

$$\lfloor 10 \div 3 \rfloor = \lfloor 3\frac{1}{3} \rfloor = 3$$

Suppose that I gave it to you with no brackets: $\square \div 3 = 4$

What is the answer to $6 \div 3 = \underline{\quad}$
 $15 \div 3 = \underline{\quad}$
 $18 \div 3 = \underline{\quad}$
 $9 \div 3 = \underline{\quad}$
 $3 \div 3 = \underline{\quad}$
 $\square \div 3 = 4 \quad (12)$

So 4 is one of the numbers that works. I'll put a dot on 4."

Are there other numbers? $(12\frac{1}{2})$

What did I do wrong? ("Put the dot on 4 instead of on the 12.")

What else works? $(12\frac{1}{3}; \text{any number that has 12; 12 and all numbers not counting 13.})$

Any more numbers? (13)

$\left[4\frac{1}{3}\right] = 4$, so 13 works. Any others? (14)

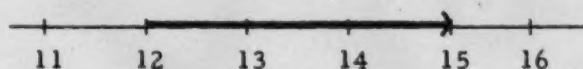
Tell me an interesting number that does not work.

(15)

Why is it interesting?

("When it's divided by 3, it comes out 5 instead of 4." "That's just a little bit over the highest." "You can go from 12 up to 15 but not 15.")

Here is a picture of the numbers that work for $\left[\square \div 3\right] = 4$:



What you have said is exactly right except for one little detail. What is the biggest number that works?

$(14\frac{9,999}{1,000,000})$

That's a pretty small fraction.

$(14\frac{999,999}{1,000,000})$

("There is no biggest number. They keep on going.")

Get me one bigger than that.

$(14\frac{9,999,999,999}{10,000,000,000})$

$$\lfloor \square \div 5 \rfloor = 1\frac{1}{10}$$

(Impossible)

Convince us that it is not possible.

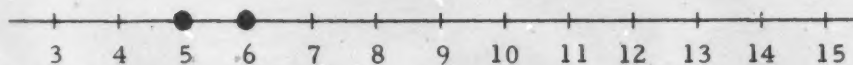
("When you put a number in there, and it comes out $1\frac{1}{10}$, it will only be 1 because in square brackets you take off fractions.")

$$\lfloor \square \div 5 \rfloor = 1 \quad \text{Tell me a number that works.}$$

(5)

What else?

(6)



A number that does not work.

(11)

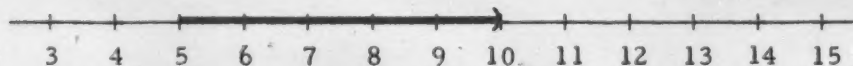
Who knows all the numbers that do work?

("All the numbers from 0 to 9.")

("From 5 to 9.")

Can you tell us a number that works that we haven't got yet?

("All the way up to 10, but not 10.")

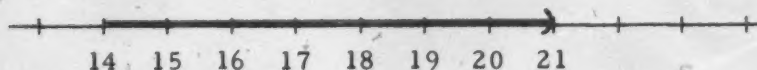


$\lfloor \square \div 7 \rfloor = 2$ Will I get numbers that are bigger than 14 or smaller than 14?

(Bigger)

Teacher puts 14 at the far left end of the number line.

("Any number from 14 up until 21, not including 21.")



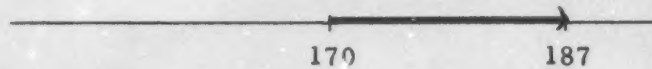
Last problem: $\lfloor \square \div 17 \rfloor = 10$ (170)

Any others?

("All the way up to 187 not including 187.")

How did you figure 187?

("170 plus 17 is 187.")



Supplement
Graphing Number Line Jumping Rules
Part One
by Edward Esty

I.

As you know, if we start at 5 and make one jump with the rule $\square \rightarrow 3 \times \square - 7$, we land on 8. We would show this jump on a number line like this:

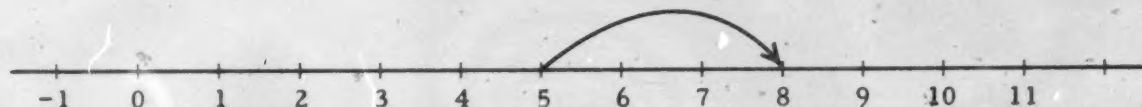


Fig. 1

A problem arises with drawing lots of jumps on the line when the jumping rule is more complicated: frequently the picture is very hard to interpret. Here is a picture of many jumps taken with the rule $\square \rightarrow \square \times \square - 7$.

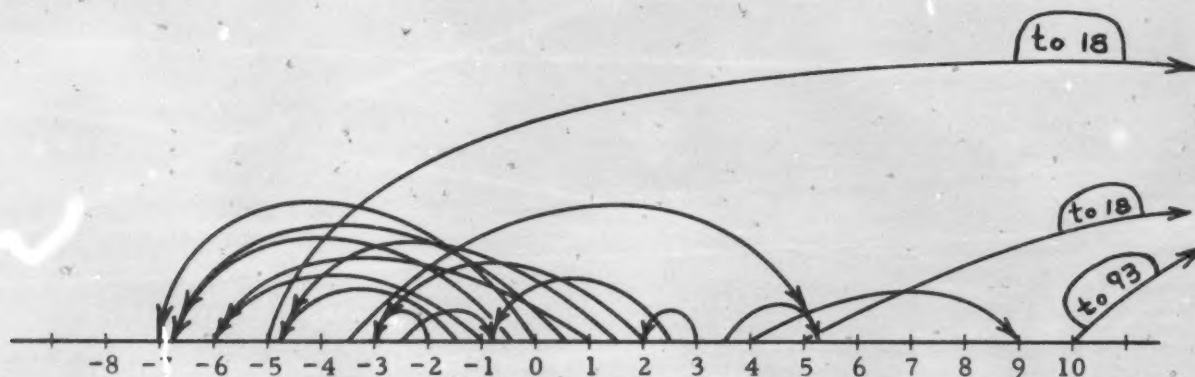
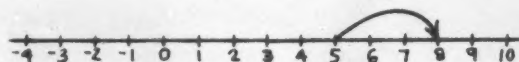


Fig. 2

In this picture it is hard to see what's going on.

Another way of showing starting and landing numbers is as follows: If we want to show a jump from 5 to 8, for example, we place a dot above 5 on the number line—a dot 8 units above 5. So instead of drawing



we draw a picture like this:

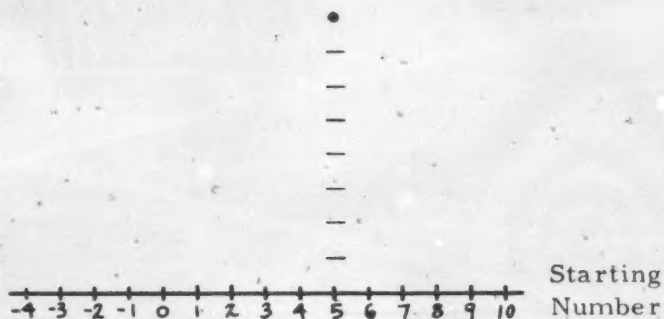


Fig. 3

To make it easier to put the dot at the correct height we make a vertical number line through zero and draw lighter lines vertically and horizontally as guides:

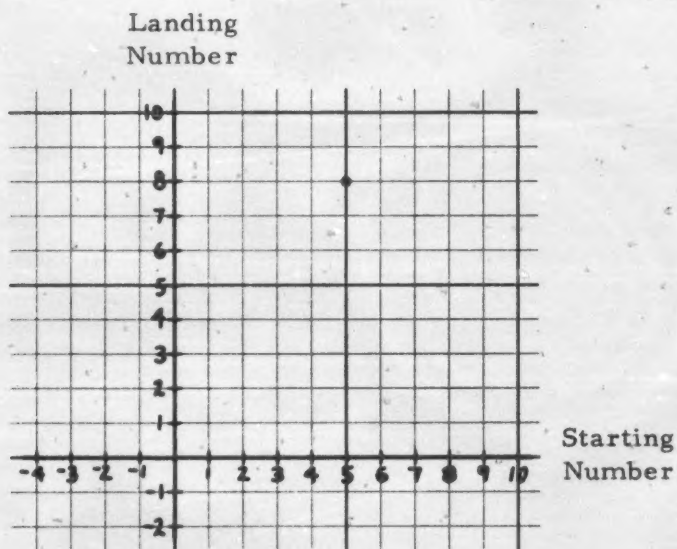
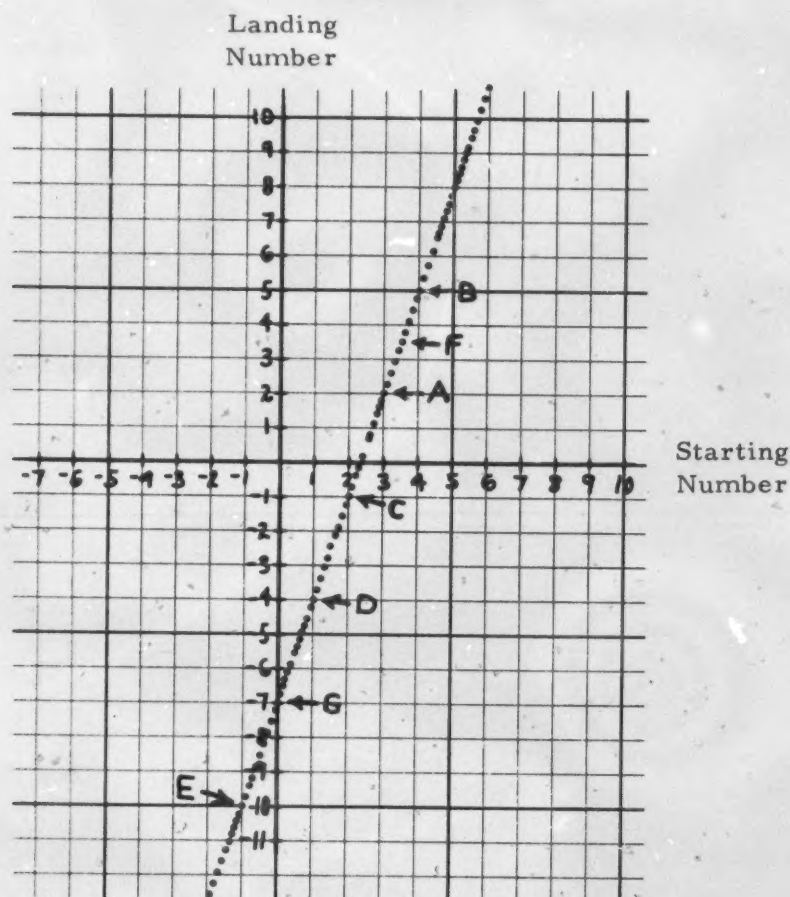


Fig. 4

The single dot in Figure 4 stands for a jump from 5 to 8. Thus the arrow of Figure 1 has been replaced by the dot of Figure 4.

The next picture shows many dots, each one of which stands for a jump made with the rule $\square \rightarrow 3 \times \square - 7$:



II.

New rule: $\square \rightarrow 2 \times \square - 7$

- In the picture below, two jumps of this rule have been shown. One is the jump from 5 to 3. What is the other? From _____ to _____.

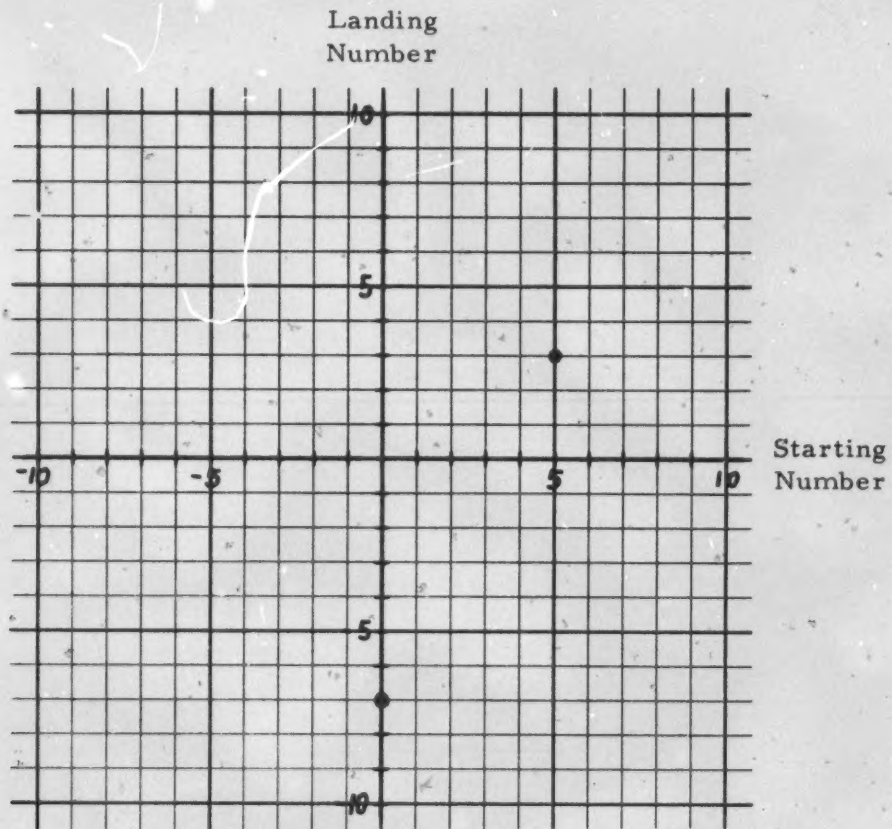


Fig. 6

- With this rule, if you start at 3, where do you land? _____ Find 3 on the starting number line, move up or down the number of spaces corresponding to your answer, and put a dot there. You have pictured the jump starting at 3.

3. Continuing with the rule $\square \rightarrow 2 \times \square - 7$, where do you have to start in order to land on 0? _____ Draw a dot in Figure 6 corresponding to this jump. It will be on the starting number line because after you find the starting number you don't go up or down at all in order to show the landing number.
4. What is the standstill point for this rule? _____ Draw the dot which shows the jump from the standstill point.

III.

New rule: $\square \rightarrow 2 \times \square - 3$

Fill in the table:

	Start	Land
1.	0	
2.		0
3.	4	
4.	-3	
5.		

← Standstill Point

Go back to Figure 6 and draw dots showing each of these jumps.

IV.

Rule: $\square \rightarrow \frac{1}{3} \times \square - 7$

1. In Figure 7 below, draw the dot for the jump that starts at 12.
2. Draw the dot for the jump that lands on -8.
3. Draw dots for at least five other jumps with this rule. Try to find fast ways of doing this.

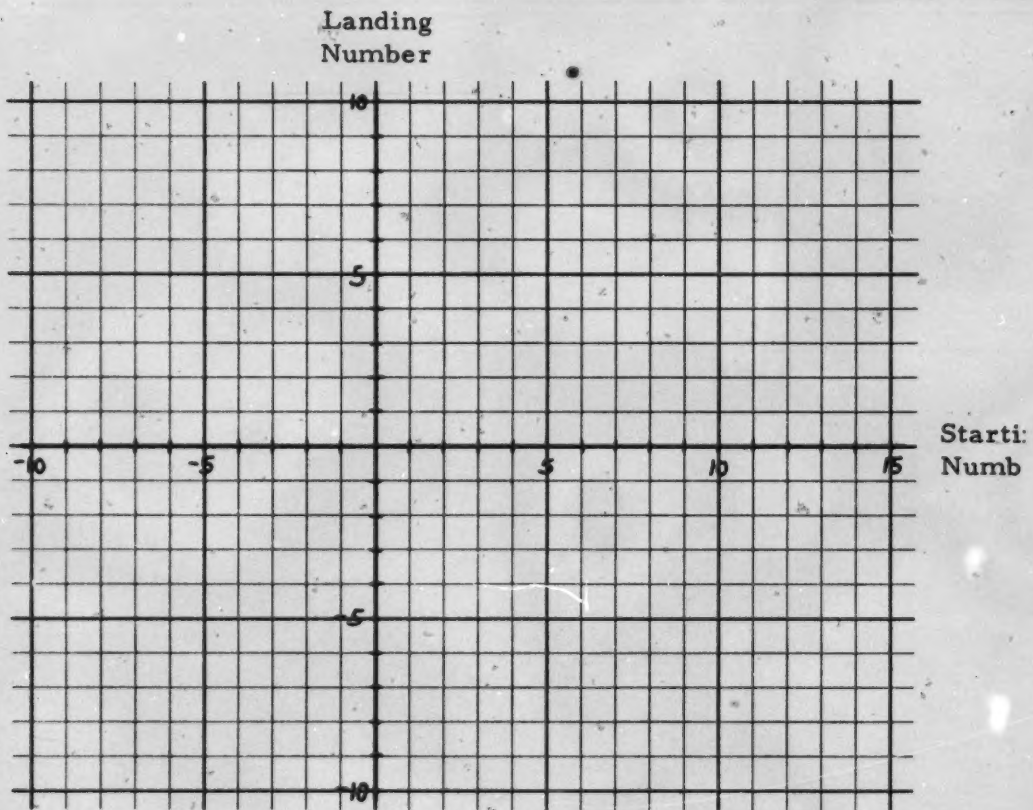


Fig. 7

The preceding sections suggest that for each rule all the jumps can be shown by a straight line. This is in fact the case if you consider the lines going on indefinitely in both directions. The lines show countless jumps.

V.

Circle the appropriate jumping rule for each of these pictures:

1.

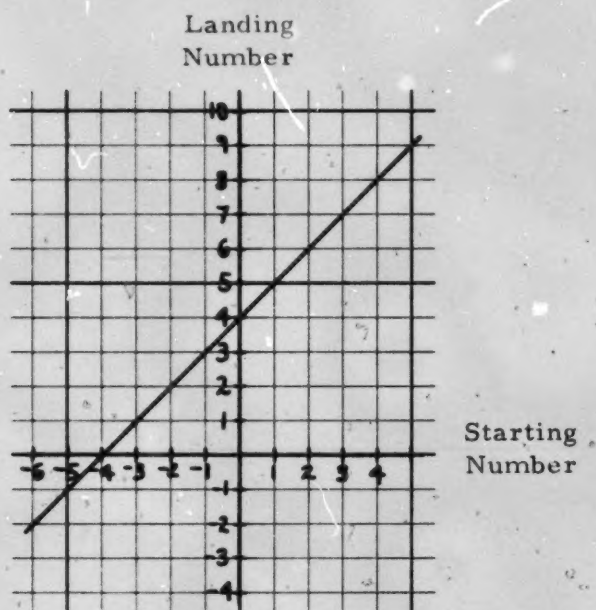


Fig. 8

$$\square \rightarrow \square$$

$$\square \rightarrow 4 \times \square$$

$$\square \rightarrow \square - 4$$

$$\square \rightarrow \square + 4$$

$$\square \rightarrow \square + 5$$

2.

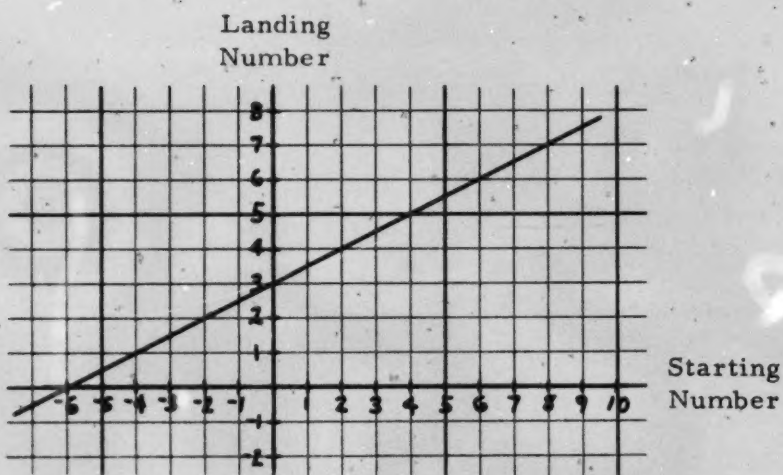


Fig. 9

$$\square \rightarrow \frac{1}{2} \times \square + 3$$

$$\square \rightarrow 2 \times \square + 3$$

$$\square \rightarrow \square \times \square + 3$$

$$\square \rightarrow 3 - \square$$

3. Landing Number

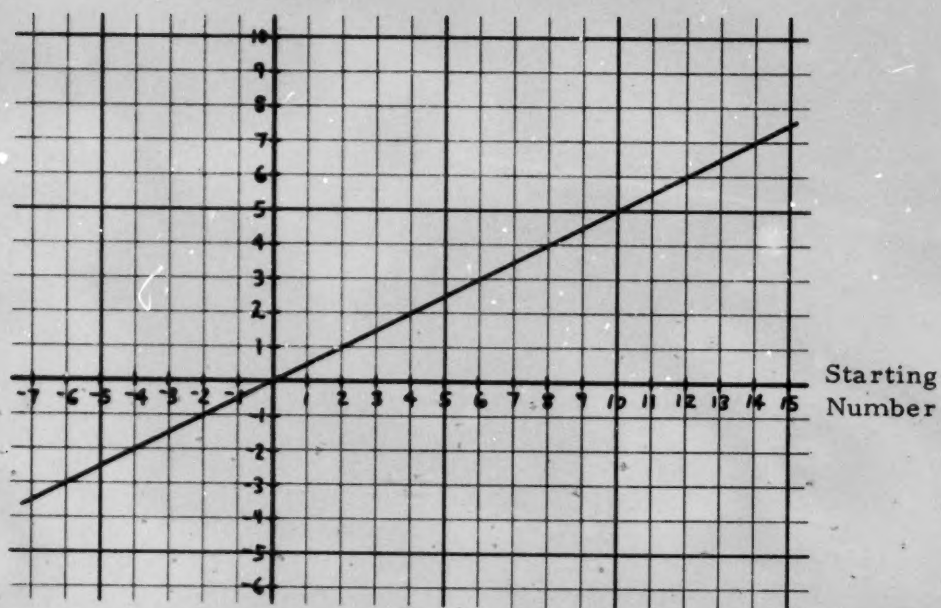


Fig. 10

$$\square \rightarrow \square + \frac{1}{2}$$

$$\square \rightarrow \square + 2$$

$$\square \rightarrow 2 \times \square$$

$$\square \rightarrow \frac{1}{2} \times \square$$

$$\square \rightarrow \frac{\square}{2} - 2$$

4. Landing Number

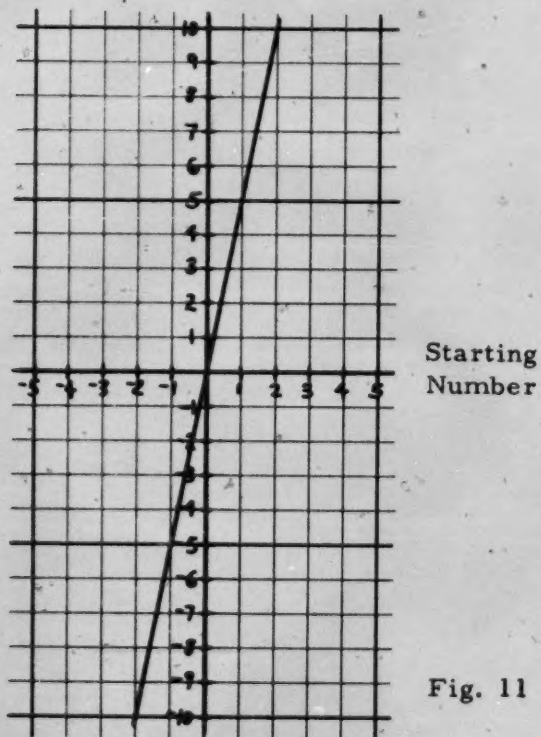


Fig. 11

$$\square \rightarrow \frac{\square}{5}$$

$$\square \rightarrow \square + 4$$

$$\square \rightarrow 5 \times \square$$

$$\square \rightarrow \square$$

VI.

The dots in the next picture show two of the jumps of the rule $\square \rightarrow \square \times \square - 7$. Predict where dots for other jumps will be; then draw dots for at least three more jumps that you compute from the rule. Include the jump that starts at 0, but do not try to draw the dot for the jump that lands on 0.

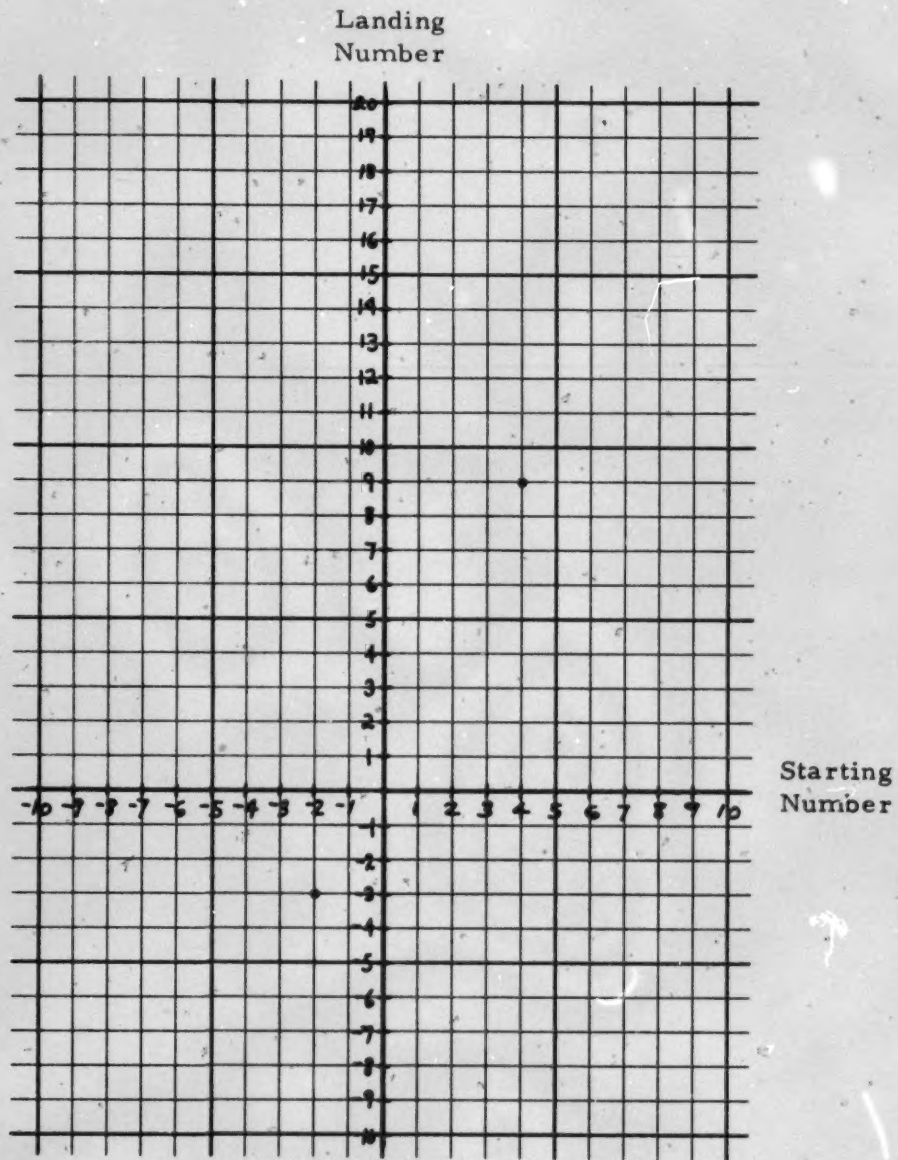


Fig. 12

After you have shown several jumps on the preceding page, see page 28 for a picture of more jumps with this rule.

VII.

Circle the appropriate jumping rules.

1.

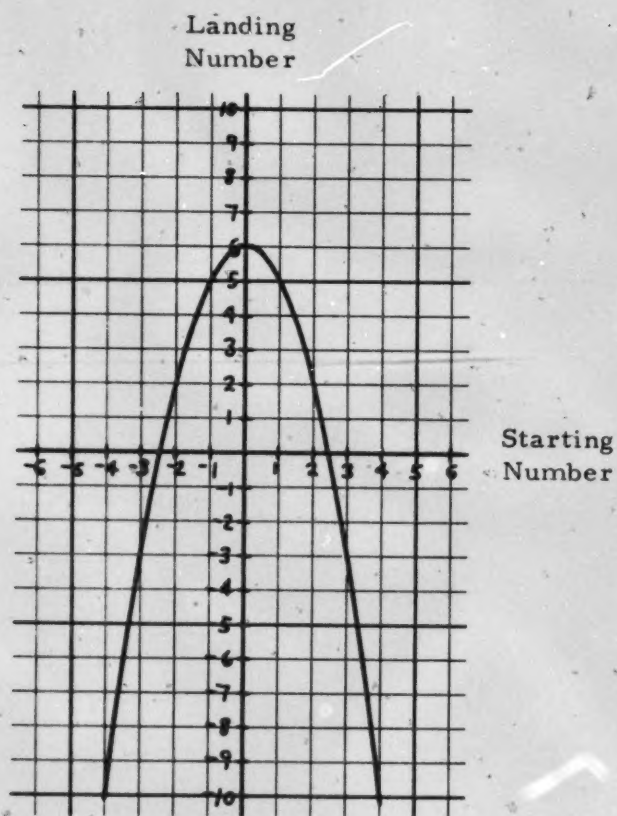


Fig. 13

$$\square \rightarrow (6 - \square) \times \square$$

$$\square \rightarrow 6 + \square \times \square$$

$$\square \rightarrow 6 - \square$$

$$\square \rightarrow 6 - \square \times \square$$

2.

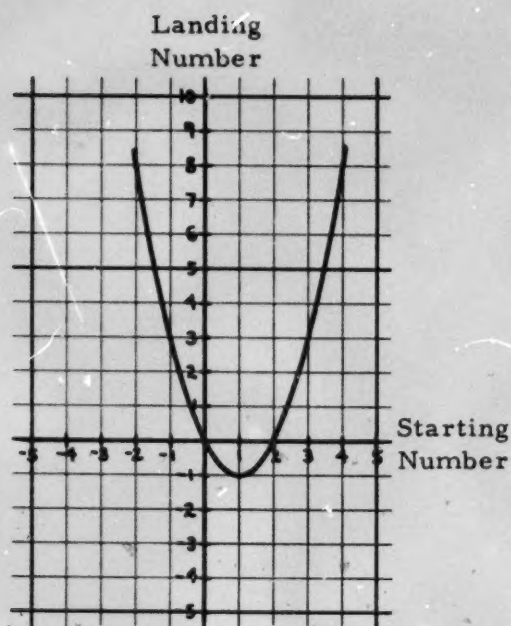


Fig. 14

$$\square \rightarrow \square \times \square - 2 \times \square$$

$$\square \rightarrow \square \times \square$$

$$\square \rightarrow \square \times \square - 2$$

$$\square \rightarrow 3 \times \square - 6$$

3.

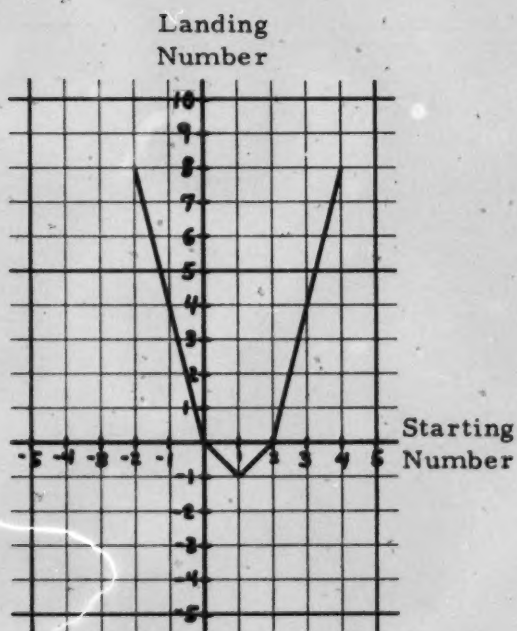


Fig. 15

$$\square \rightarrow \square \times \square - 2 \times \square$$

$$\square \rightarrow \square \times \square$$

$$\square \rightarrow \square \times \square - 2$$

$$\square \rightarrow 3 \times \square - 6$$

None of the above

(A very common mistake that children make is to draw a few dots and connect them with straight line segments. The graph of $\square \rightarrow \square \times \square - 2 \times \square$ is shown in Fig. 14, not in Fig. 15.)

VIII.

Here are the graphs (or pictures of the jumps) of these two rules:

$$\square \xrightarrow{A} 3 \times \square - 7$$

$$\square \xrightarrow{B} \square \times \square - 7$$

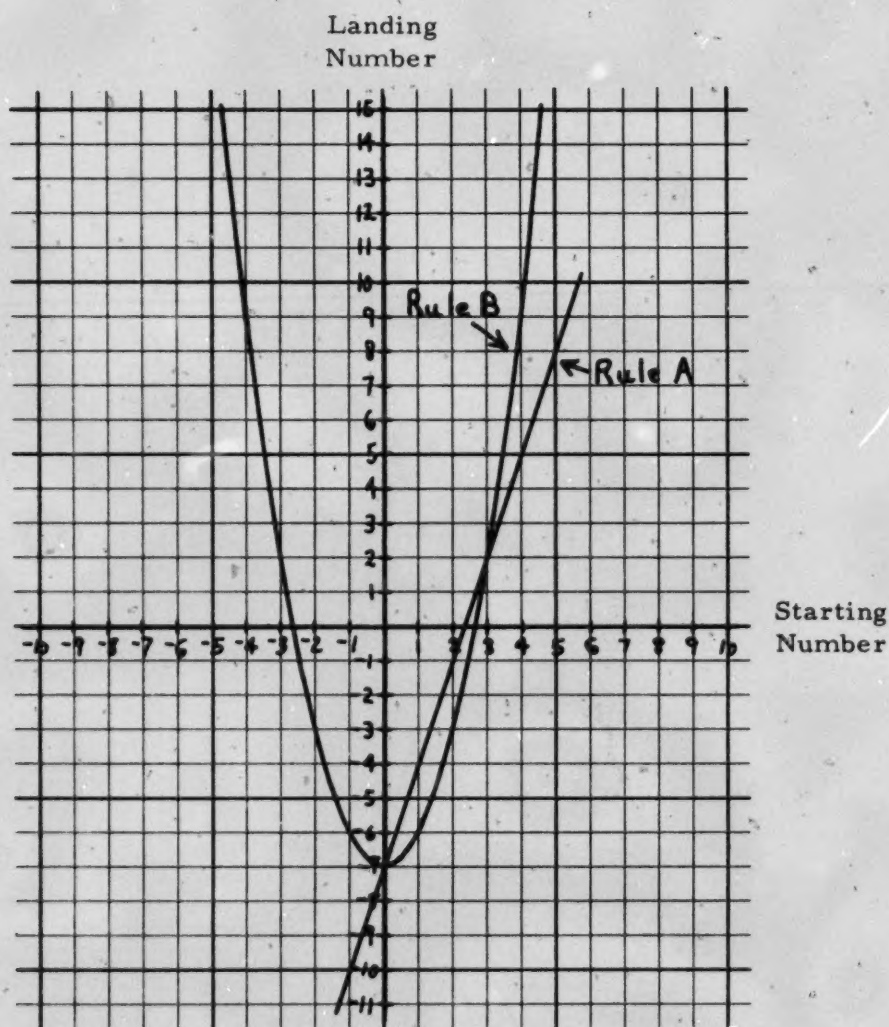


Fig. 16

Answer the following, if you can, just by looking at the graph on the preceding page, rather than by computing.

1. You can see that the rules tie (yield the same jump) starting at 3, because the graphs cross directly above 3 on the starting number line. At what other starting point do the rules tie? _____
2. If you start on a number bigger than 3, which rule wins, A or B? _____
3. Describe all the numbers where rule A wins.
4. Find two starting numbers where rule A wins by 2.
_____, _____
5. What is the smallest landing number obtainable using rule B? _____

(If you are in doubt about any of your answers you can check them by referring to the rules and computing.)

IX.

The picture below shows the graphs of four different jumping rules, which we will call C, D, E and F. (We won't write them out explicitly.)

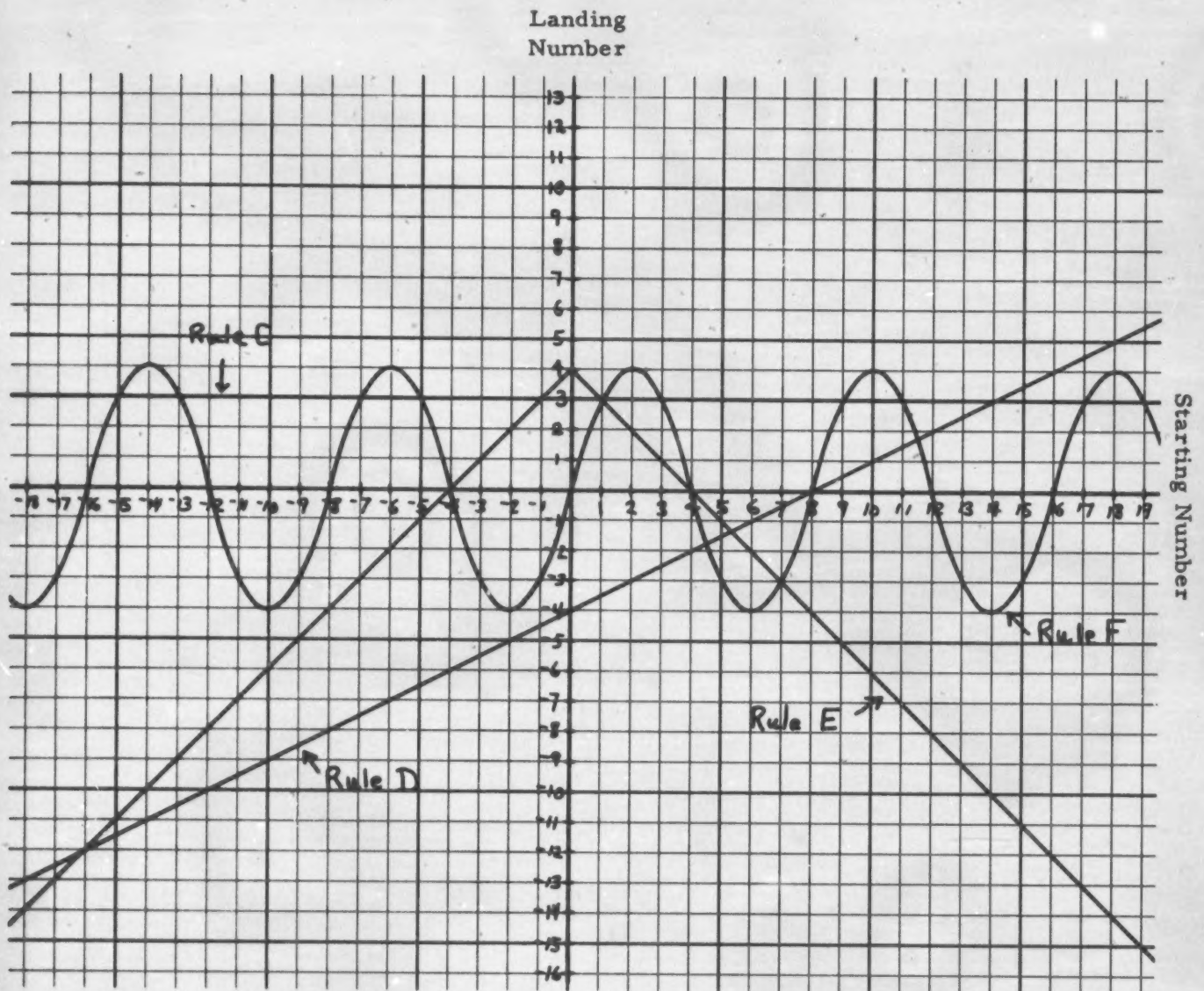


Fig. 17

1. Starting at 2 where do you land using rule F ? _____ Using rule C ? _____ Rule E ? _____ Rule D ? _____ Where should you start so that rules C and D tie? _____ At how many different starting numbers do E and F tie? _____ Name these starting numbers.

2. There is a number to start on so that three of the four rules tie. What is the starting number and what are the three rules?
Starting number: _____ Rules: _____ , _____ , _____
3. Give two starting numbers larger than 100 where rules C and F tie.
_____ and _____
4. If we start at 10, rule F yields the largest landing number (4), rule C gives the next largest (3), rule D is next (1) and rule E gives the smallest (-6).
Starting at 6, which rule wins? _____ Which rule gives the next largest landing number? _____ Next? _____ Smallest landing number? _____
5. Find a starting number so that the landing numbers, from the largest to smallest, are given by rules D , C , F and E in that order. _____
6. Find another starting number so that the order of the rules is C , E , D , F .

7. How might a child explain why the order E , C , D , F is impossible?

This page may be removed
for reference if desired.

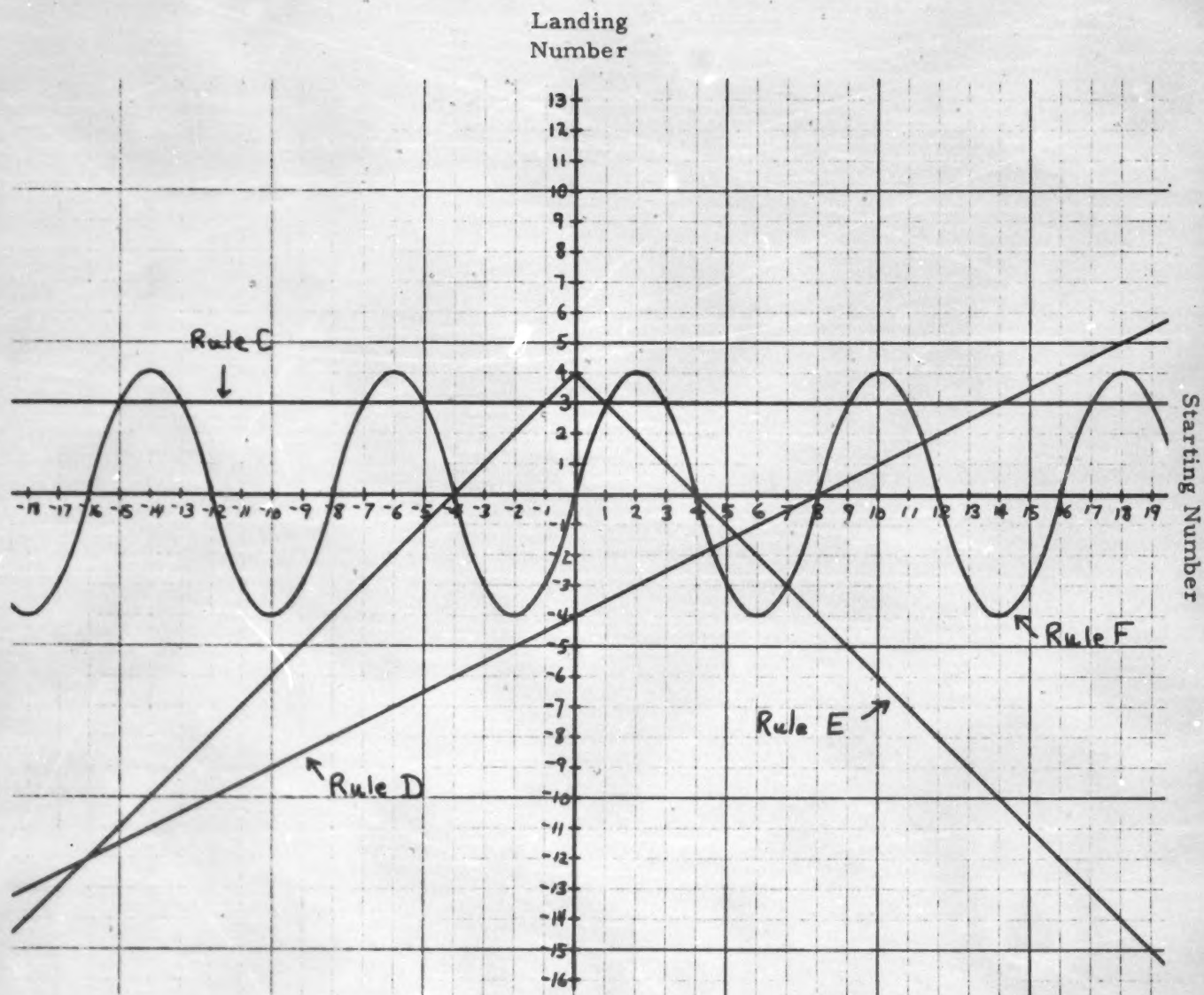


Fig. 17

8. Find another impossible order. _____
9. If we start at 12 and use rule F, we land on 0. (Notice that 12 is therefore not a standstill point for rule F.) Name two starting numbers which give jumps to 0 using rule E. _____ and _____ Name all the starting numbers which give jumps to 0 using rule D. _____ How many such numbers are there for rule C? _____ How many for rule F? _____
10. Put a dot on Figure 17 showing a jump from 12 to 12. Note that none of the rules C, D, E or F gives a jump from 12 to 12. Now put dots on Figure 17 showing jumps from 11 to 11, 10 to 10, 9 to 9, -9 to -9, -2 to -2, and 2 to 2. One of the dots you have drawn shows a standstill point for one of the rules. Which rule? _____ Rule F has three standstill points. What are they? _____, _____, _____ Find all the standstill points for rule C. _____ Find all the standstill points for rule D. _____
11. If the jump from $\frac{1}{2}$ to $3\frac{1}{2}$ (given by rule E) were drawn on a number line the jump would go to the right. The jump from 3 to 1 goes to the left. What are all the starting numbers which give jumps to the left, using rule E?

12. Describe all the starting numbers which give jumps to the right, using rule F.

Thirty jumps with the rule $\square \rightarrow \square \times \square - 7$:

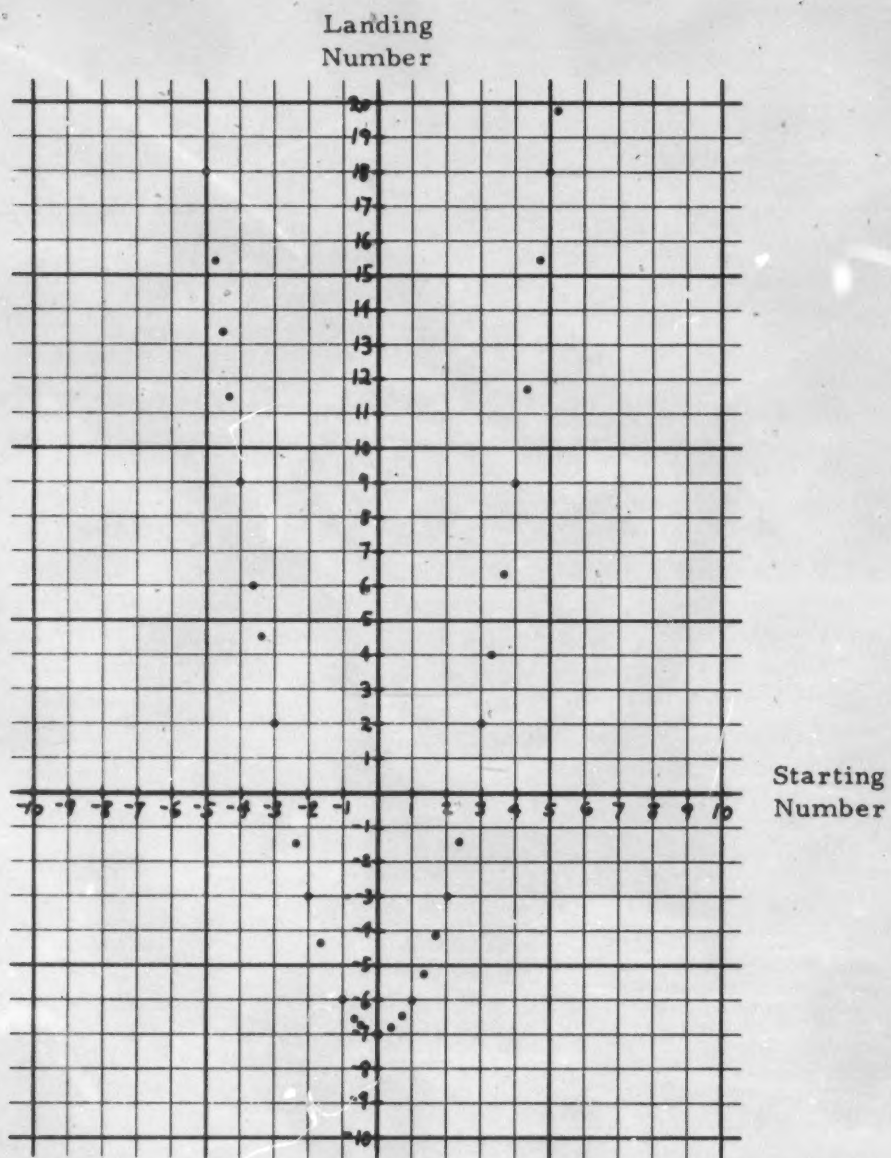


Fig. 18

Draw the curve smoothly connecting these dots to see all the jumps with this rule that can be shown here.

X. Here is the graph of a jumping rule:

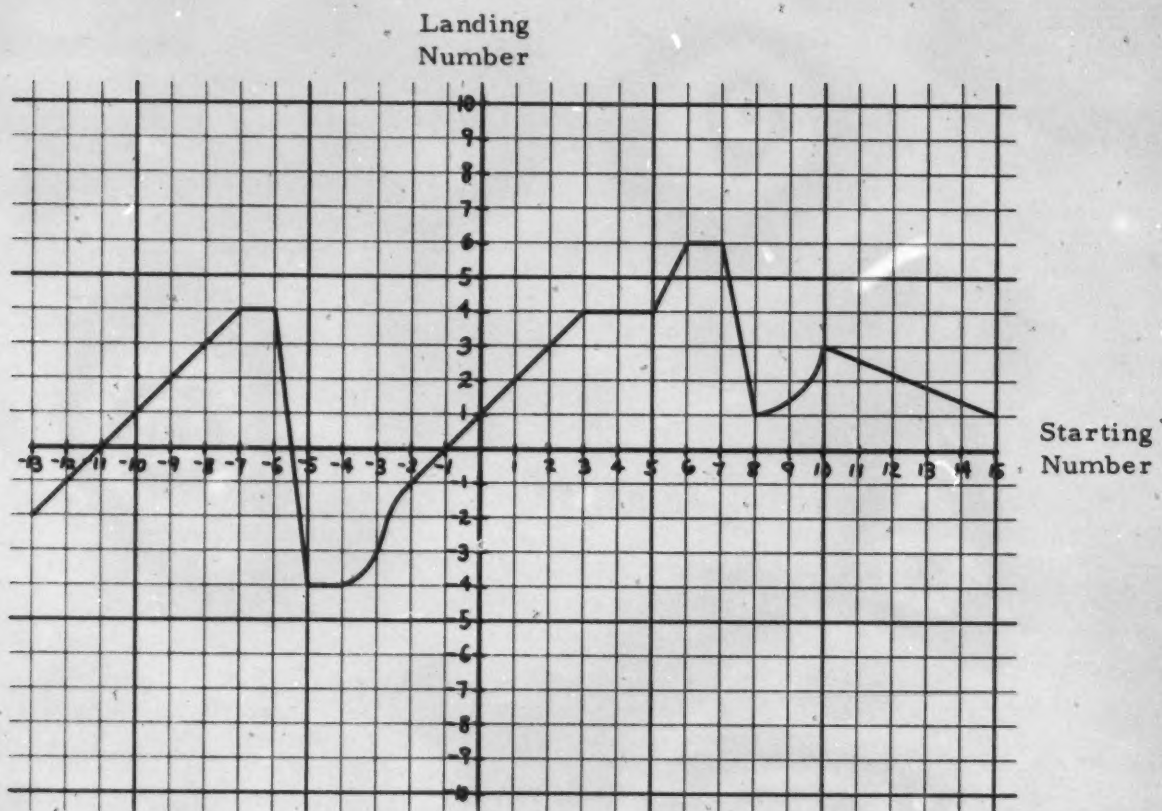


Fig. 19

(Only jumps starting from numbers between -13 and 15, inclusive, are shown.)

The rule that gives this graph is so complicated that we won't bother to write it down explicitly. We'll abbreviate it by writing it as $\square \rightarrow \text{wiggly}$ ("Box goes to squiggle"), and we'll call it rule A. That is, the graph above is the graph of the rule $\square \xrightarrow{A} \text{wiggly}$.

1. Start at 2 and make one jump with rule A. Land? _____
2. Name three starting points which will give jumps landing at 4.
_____ , _____ , _____
3. Name some starting numbers which will give jumps to negative numbers.

4. Name as many standstill points as you can for rule A . _____

5. Find the places where the landing point is twice the starting point.

6. Suppose we start at -9 and make three consecutive jumps with rule A .
What is the final landing point? _____
7. Now we consider a different rule, B , which is $\square \xrightarrow{B} \text{wavy line} + 2$.
Since rule A gave a jump from 10 to 3 , rule B gives a jump from 10
to $3 + 2$, or 5 . Put a dot on Figure 19 showing the jump from 10 to 5
given by rule B . Start at 6 and make one jump with rule B . Land?
_____ Put a dot on Figure 19 to show this jump. Put a dot on
Figure 19 to show the jump from -4 using rule B . Now draw the
graph of rule B on Figure 19 .
8. Describe in words what the graph of the rule $\square \xrightarrow{C} \text{wavy line} - 100$
would look like.

XI.

Here is the graph of the rule $\square \xrightarrow{D} \frac{1}{2} \times \square \times \square - \square + 3$:

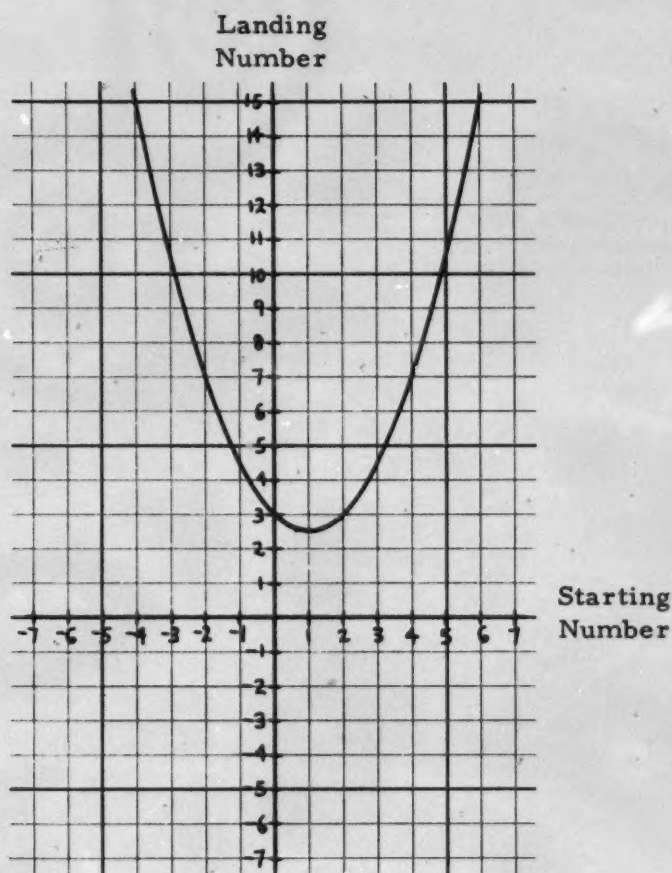


Fig. 20

- On Figure 20 draw the graph of the rule $\square \xrightarrow{E} \frac{1}{2} \times \square \times \square - \square + 5$.
(A likely wrong answer here is to copy the graph already given 5 units higher.)
- Write a rule whose graph is exactly the same shape as rule D's graph, but which has a jump from -2 to 0:
 $\square \xrightarrow{F} \underline{\hspace{4cm}}$
- What is the smallest landing number obtainable with rule F?

XII.

You have seen that if two jumping rules are alike except for a number added on at the end, then their graphs will be the same shape but one will be higher than the other. Graphing $\square \rightarrow 3 \times \square + 4$ will be simple if we know how to graph $\square \rightarrow 3 \times \square$. Graphing $\square \rightarrow -\frac{1}{4} \times \square - 7\frac{1}{2}$ will be easy if we know how to graph $\square \rightarrow -\frac{1}{4} \times \square$.

Here is the graph of $\square \rightarrow 3 \times \square$:

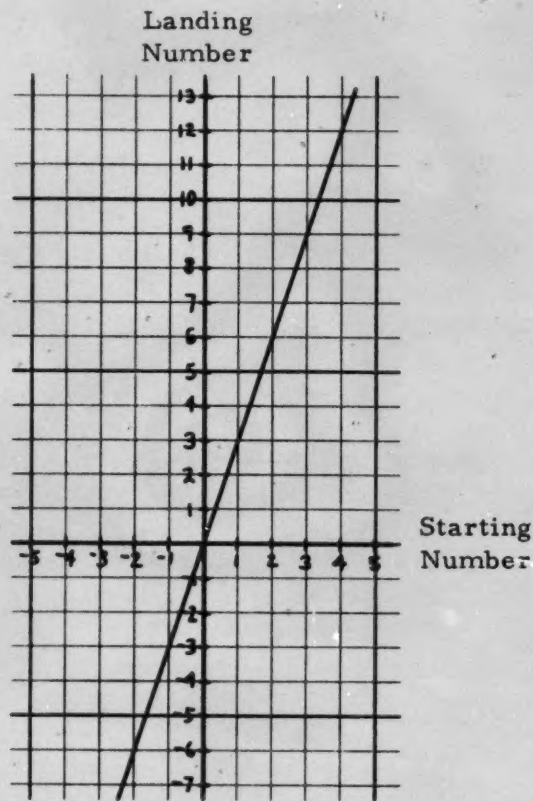


Fig. 21

1. Draw the graph of $\square \rightarrow 3 \times \square + 4$ on Figure 21.

This is the graph of .

$$\square \rightarrow -\frac{1}{4} \times \square$$

2. Draw the graph of

$$\square \rightarrow -\frac{1}{4} \times \square - 7\frac{1}{2}$$

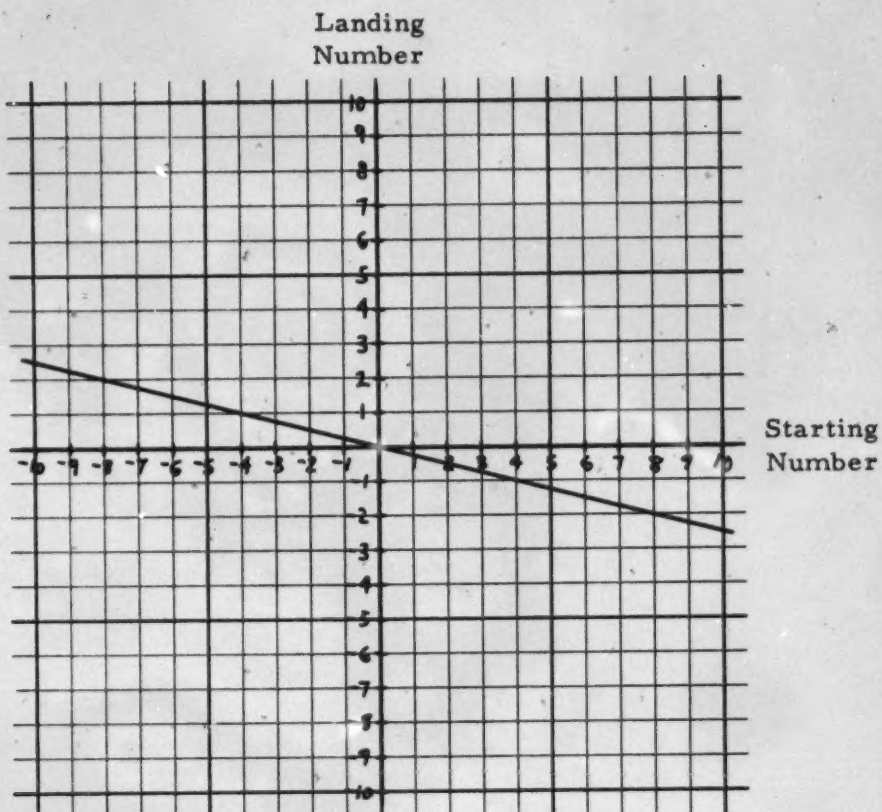


Fig. 22

3. Graph the rule

$$\square \rightarrow \frac{1}{2} \times \square$$

4. Then graph the rule

$$\square \rightarrow \frac{1}{2} \times \square - 1$$

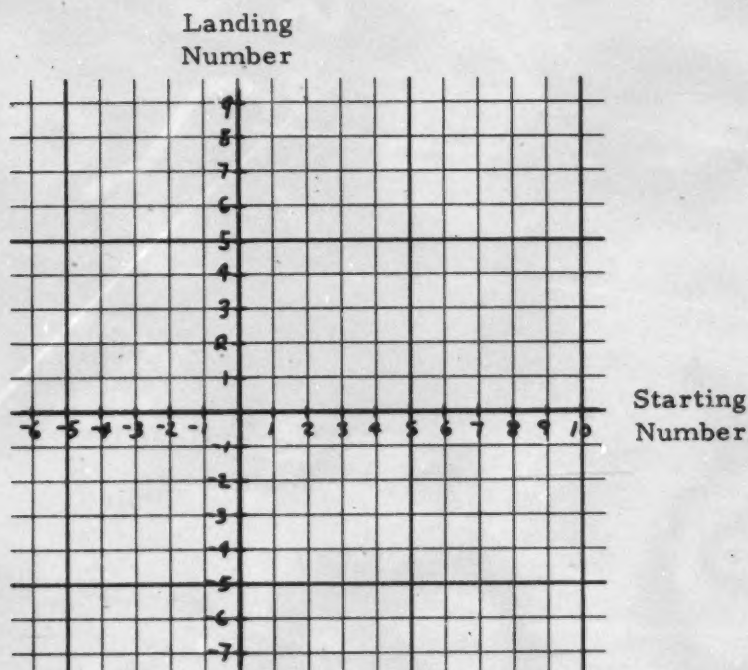


Fig. 23

XIII.

Write the rules which give these graphs. The first one is done for you.

1. Rule G:

$$\square \rightarrow \underline{-\frac{1}{2} \times \square + 5}$$

2. Rule H:

$$\square \rightarrow \underline{\hspace{2cm}}$$

3. Rule J:

$$\square \rightarrow \underline{\hspace{2cm}}$$

4. Rule K:

$$\square \rightarrow \underline{\hspace{2cm}}$$

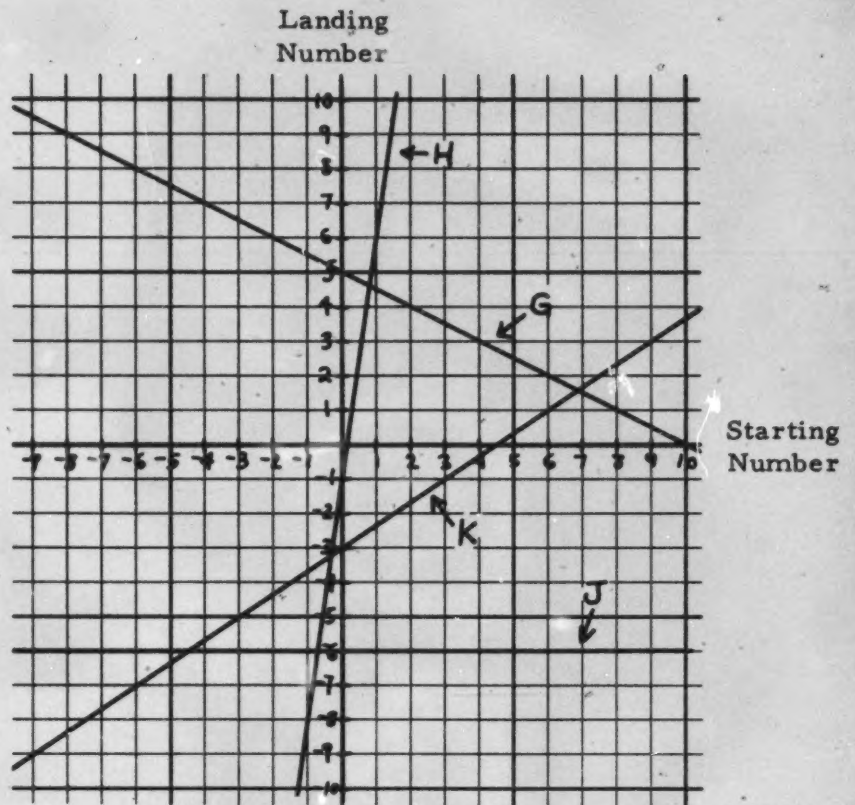


Fig. 24

Teachers (and children) frequently wonder why the graphs of some rules are straight lines while other rules give curves. One way of looking at $\square \rightarrow 3 \times \square$ is to say that as the starting number increases the landing number increases three times as fast. It is the fact that the rate of increase is a constant number (3) that makes the graph a straight line. Adding or subtracting a number at the end does not destroy the straightness—it just moves the line up or down.

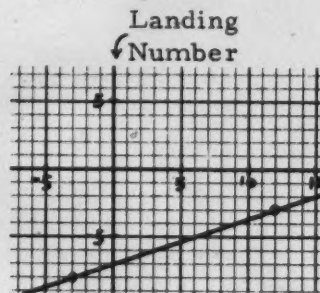
Answers

- I. 1. 4 to 5 4. -1 to -10
 2. 2 to -1 5. $3\frac{1}{2}$ to $3\frac{1}{2}$
 3. 1 to -4 6. 0 to -7

- II. 1. 0 to -7 3. $3\frac{1}{2}$
 2. -1 4. 7

- III. 1. -3 4. -9
 2. $1\frac{1}{2}$ 5. The standstill point is 3.
 3. 5

- IV. The dots will fall on the line shown.



- V. 1. $\square \rightarrow \square + 4$ 3. $\square \rightarrow \frac{1}{2} \times \square$
 2. $\square \rightarrow \frac{1}{2} \times \square + 3$ 4. $\square \rightarrow 5 \times \square$

- VI. See page 28.

- VII. 1. $\square \rightarrow 6 - \square \times \square$
 2. $\square \rightarrow \square \times \square - 2 \times \square$
 3. "None of the above"

- VIII. 1. 0
 2. B
 3. All numbers greater than 0 and less than 3.
 4. 1 and 2
 5. -7

IX. 1. The answers, in order, are:

4, 3, 2, -3 ; at 14 ; 4 ; -4, 1, 4 and 7

2. 1 ; C, E, and F

3. Any multiple of 8 plus 1 or plus 3, such as 105 or 107.

4. C ; D ; E ; F

5. Any number greater than 14 and less than 17. Also, any number greater than 17 that is less than 3 spaces away from any multiple of 8 plus 6; that is, numbers from 19 to 25; from 27 to 33, from 35 to 41, etc.

6. Any number greater than $4\frac{1}{2}$ and less than $5\frac{1}{3}$.

7. E, C, D, F is impossible because E wins only between -1 and 1, and in that region F always beats D.

8. Other impossible orders are:

D, C, E, F	E, D, C, F	F, D, C, E
D, E, C, F	E, D, F, C	F, D, E, C
D, E, F, C	E, F, C, D	F, E, C, D
D, F, E, C	E, F, D, C	F, E, D, C

9. -4 and 4 ; 8 ; none ; infinitely many

10. E ; -3, 0, 3 ; 3 ; -8

11. All numbers bigger than 2

12. All numbers that are either less than -3 or between 0 and 3 exclusive.

X. 1. 3

2. $7\frac{2}{5}$ or numbers which are between -7 and -6 (inclusive)
or between 3 and 5 (inclusive)

3. Numbers between -13 and -11 not including -11
or between $-5\frac{1}{2}$ and -1 (exclusive)

4. -4, -3, 4 and 6

5. 1

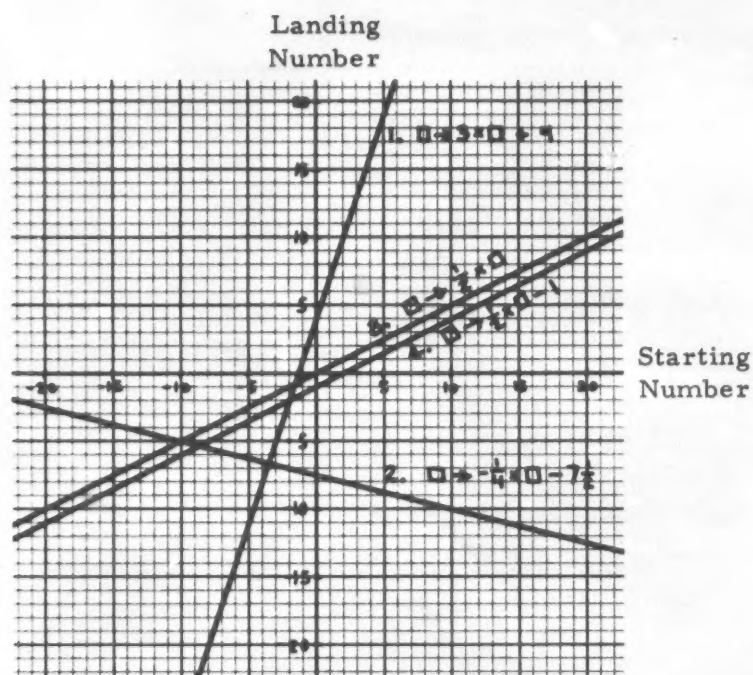
6. 4

7. $6 \xrightarrow{B} 8$. The graph of rule B looks like the graph of rule A, except that it is 2 units higher.

8. The graph of rule C looks like the graph of rule A, except that it is 100 units lower.

- XI. 1. The graph of rule E looks like the graph of rule D, except that it is 2 units higher.
2. $\square \rightarrow \frac{1}{2} \times \square \times \square - \square - 4$
3. $-4\frac{1}{2}$

XII.



XIII.

2. $\square \rightarrow 7 \times \square - 1$
3. $\square \rightarrow -6$
4. $\square \rightarrow \frac{2}{3} \times \square - 3$